

Transportation

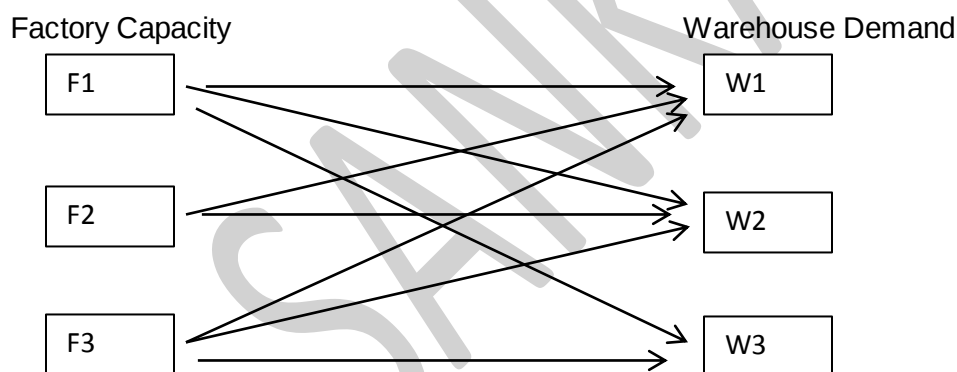
Meaning in a nutshell:-

A typical transportation problem is concerned with –

- (i) Selection of routes from various locations to warehouses, branches etc.
- (ii) Schedule the transportation in such a way so as to minimize cost.

Thumb rule = Minimize cost

Example:



We have to ensure that the demand is fulfilled at the least possible cost.

❖ **Some Concepts:**

1. Feasible solution

Total Capacity (supply) = Total Requirement (Demand)

2. Initial Basic Feasible Solution:

No of allocations = $m + n - 1$

m = No. of rows

n = No. of columns

3. Optimal Solution:

Situation where total transportation cost is minimum
Transportation

IBFS

(Initial Basic Feasible Solution)

- North – West Corner Rule
- Least Cost Method
- **Vogel's Approximation Method**
- Row Minima
- Column Minima

OST

Modified
Distribution

❖ INITIAL BASIC FEASIBLE SOLUTION

North –West Corner Rule

Select the North – West number
i.e. left most number in the matrix

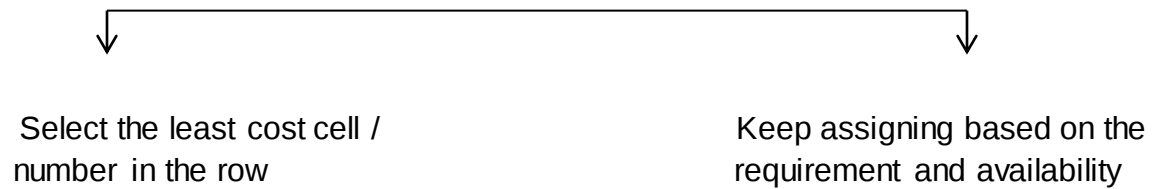
Keep assigning based on the
requirement and availability

Least Cost Method

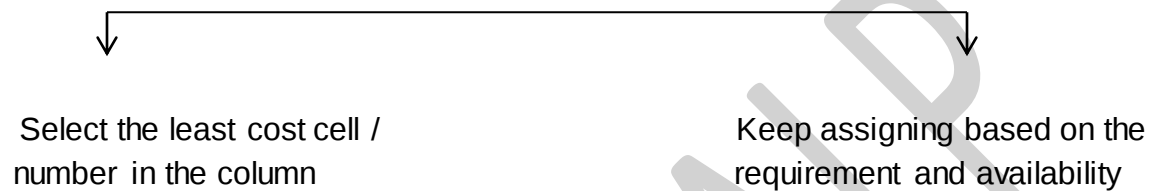
Select the least cost cell /
number in the matrix

Keep assigning based on the
requirement and availability

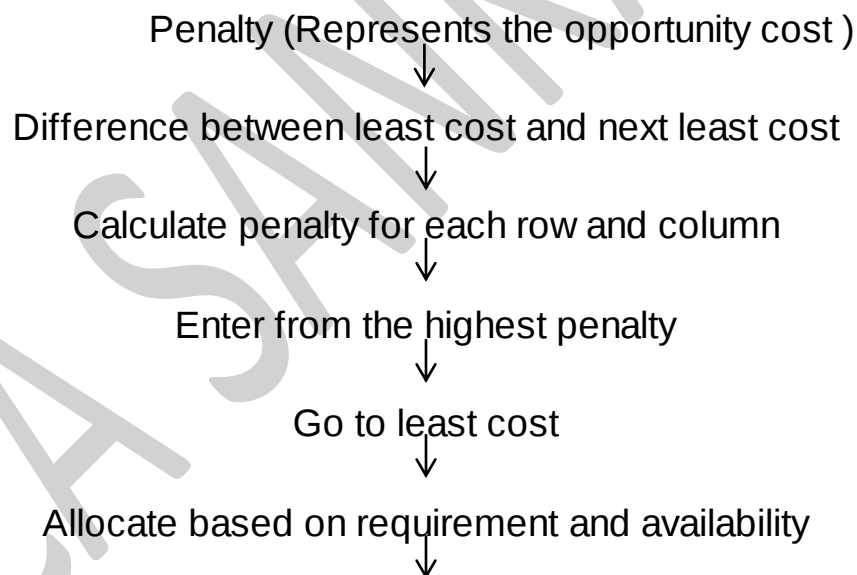
Row Minima



Column Minima



Vogel's Appropriation Method (Most Important)



- **If PENALTY is same;**
Select based on HIGHEST QUANTITY
- **If QUANTITY is same;**
Select based on LEAST COST

❖ OPTIMALITY SOLUTION TEST

1. Check for Degeneracy

i.e. No. of allocations = $m + n - 1$

where, m = no of rows

n = no of columns

2. If equal : Proceed with Allocation of values

If not equal: Remove degeneracy by Inserting (e) Epsilon at the least cost independent cell from where loop is not possible.

✓ Rules of loop

- Start from the cell
- Go horizontal / vertical
- Turn only at allocations
- Reach back from where started

3. If no of allocations = $m + n - 1$ proceed with, Allocations of values,

✓ How to allocate values :

- Allocate 0 in the first row or wherever there are maximum allocations
- Allocated cell cost = Row value + Column value
- Allocate values for all rows and columns

Find out Net Evaluation

NE = Unallocated cell cost – (Row value + Column value)

If NE = 0 or POSITIVE

We have reached optimality

If NE = Negative

- a. Select cell with least number.
- b. Draw a loop from that cell [Follow rules of loop].
- c. Put alternating signs in the cell on its route **starting with +** and changing signs at turns.
- d. Make adjustments.

Again proceed with checking for degeneracy.

❖ Concept of Alternative Solution:

If in the final solution:

- Net Evaluation is 0



Means Alternative solution exist



To find the new solution



Draw a loop from the cell whose net evaluation is 0



Make adjustments



New alternative solution is formed.

❖ Some points to remember :-

1. **Maximization type Matrix :** (Revenue , Income , Sales)

In case of maximization question



Convert the matrix into minimization type



For that, subtract all the numbers in the matrix from the highest number and from itself.



However, in the final solution we will substitute the given solution with the original maximized matrix.

2. In case of Net Evaluations, if there is more than one highest negative number then in such a case, we will draw 2 loops from both the highest negative numbers, check the least negative sign allocations for both loops and select that loop which gives the least allocations.
 3. Epsilon means Dummy Allocation.
 4. In case of unbalanced matrix i.e Requirement = Availability ,
Add a dummy row or column as the case may be.
- Note : The cells of the dummy row or column will always be denoted by 0.**
5. **Prohibited Routes** : Cannot allocate any resource in that cell.
Denoted by M.
 6. Unique Solution : If Net Evaluation is 0 then unique solution exists.
 7. Net Evaluation is always 0 for allocated cell.

SIMPLEX

Introduction : -

It was developed in 1947 by Mr. George Dantzig which arose out of the limitation of Graphical method.

❖ Steps and Concepts of Simplex: -

In simplex, we will always try and complete 3 initial steps:

- (i) Restate the equation
 - (ii) Equalize the inequalities
 - (iii) Identity matrix
-
- If there is a $\leq$ sign in an equation, we will **ADD** slack variable represented as **(S)** (spare capacity).
 - If there is a $\geq$ sign in an equation, we will **SUBTRACT** the surplus variable **(S)** from it and add **(A) Artificial variable** to represent the identity matrix.
 - The coefficient for slack & surplus variable in the objective function will always be 0, whereas coefficient of Artificial Variable (A) will be represented as M.
(M) = Exorbitantly high number.
 - The coefficient of A in a maximization sum is **-M** and in minimization sum is **+M**.

❖ Optimality

For maximization type problem we will reach optimality when **ALL NER is negative or zero.**

For minimization type problem we will reach optimality when **ALL NER is positive or zero.**

- Now let us understand step by step procedure of Simplex

Step 1 – Convert the given matrix by adding a slack variable or subtracting surplus variable.

However, we have to ensure that in the revised matrix

“AN IDENTITY MATRIX IS FORMED”.

2 Tier	3 Tier	4 Tier
1 0	1 0 0	1 0 0 0
0 1	0 1 0	0 1 0 0
	0 0 1	0 0 1 0
		0 0 0 1

Step 2 - In the next step, we will put the above revised matrix/ equation in a simplex compatible form called as – Simplex Table.

Profit	Cost	Program	Quantity	Variables	Replacement Ratio
↓	↓	S_1/ S_2			
Maximum	Minimum	A_1/ A_2			

In the above table, we have to ensure that the Identity Matrix is formed & the program variable will be based on that identity matrix in a vertical format.

Fill in the whole table using the values of the equation. Also, write down C_j on top of the table C_j represents contribution per unit which will be derived from the main Simplex Table.

Then, calculate the NER

$$\text{NER} = C_j - Z_j$$

C_j = Co-efficient of parent variable (Contribution p.u.)

Z_j = Program x co- efficient of variable

Then check for optimality :

1) NER for maximization

Optimal solution is reached when NER is negative or 0.

If NER consists of positive number then select the highest positive number



Make an incoming variable from that highest positive number and denote it as key column.

2) NER for minimization

Optimal solution is reached when all numbers in NER is positive or 0.

If NER consists of a negative number then select the lowest number



Make an Incoming Variable and denote it as key column.

$$RR = \frac{\text{Quantity}}{\text{Respective key column}}$$

- Select the lowest RR for both maximum and minimum and make an outgoing variable from that lowest number.
- In case, RR has any negative or infinite number then it is to be ignored.
- In case there is a tie between two numbers in a RR then select that row which has the highest key column element.
- In case of tie in key column element, select any.

In the next table, the incoming variable will be replaced by outgoing variable and the formula to calculate the new incoming variable will be

$$\frac{\text{Key Row}}{\text{Key Element}}$$

(For the new incoming variable)

Key element refers to intersection point of incoming and outgoing variables (Key row & Key column).

For the old program in the previous table it will be calculated by using the below formula.

Old program – (Key Row x Fixed Ratio)

↓

$$\frac{\text{Non-Key Element}}{\text{Key Element}}$$

Re – iterate the above process till the optimal solution is received.

Some ADDITIONAL CONCEPTS :

❖ Alternate Optimal Solution

On a close analysis of all simplex solutions we observe that the program variables in the last table where optimality is reached we see a vertical identity matrix. Also, for those final program variables the NER is always 0.

Accordingly, we can use it as a cross check.

If the NER consists of a zero for a program variable even though it is not there in the final solution it means that alternate solution exists.

If we have to calculate the alternate solution then we should make an incoming variable from that program variable and calculate the quantity column for the next Simplex table.

❖ Feasibility of Solution

If in the final program there exists an artificial variable then the solution is unfeasible. If there is no artificial variable then the solution is feasible.

❖ Degeneracy

A solution is degenerate when Quantity column has zero in the final program. If there is no zero in the final program then the solution is not degenerate.

❖ Concept of abundant & Scarce resource

The Simplex problem is solved keeping in view the optimal utilization of machine hours for all the products.

If a particular machine is reflected in the final program then it has some spare capacity available. Therefore, it is called as abundant resource whereas if it is not there in the final program then it is a case of scarce resource.

❖ Unsolvable Degeneracy

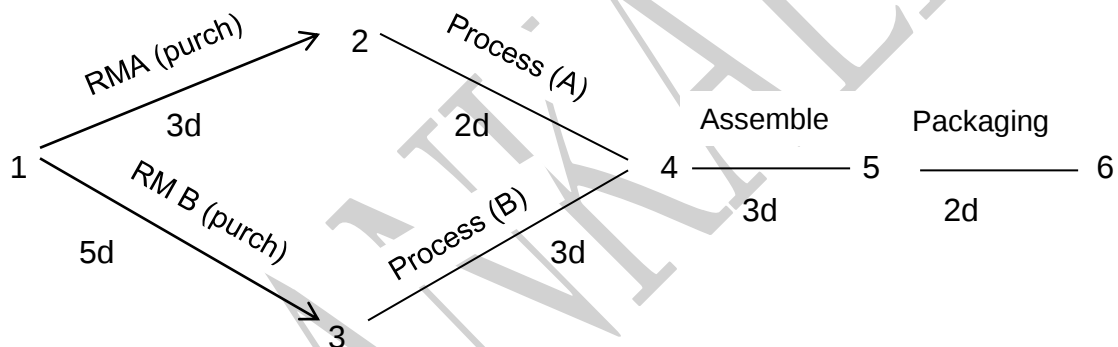
If in case, the RR has negative and infinite then it is a case of unsolvable degeneracy i.e., infinite loop which cannot be solved. In technical terms is called as unsounded unfeasible solution.

CPM - PERT

Let us take an example of making a shirt in terms of a project. Below are the activities from start to end for making a shirt.

1. Purchase raw material A = 3 days
2. Purchase raw material B = 5 days
3. Process RM A = 2 days
4. Process RM B = 2 days
5. Assemble A + B = 3 days
6. Package = 2 days

The critical path for making the above shirt using a project diagram will be as follows:



As can be seen from the above project diagram buying and processing of RM A & B is independent whereas assembling and packaging are interdependent.

In the above diagram, there are two paths for the project.

Path 1:

$$1 - 2 - 4 - 5 - 6 = 10 \text{ days}$$

Path 2:

$$1 - 3 - 4 - 5 - 6 = 13 \text{ days}$$

The longest path of the project is called as the critical path i.e. 1 – 3 – 4 – 5 – 6 days.

Buying and processing of RM A has a slack / float of 3 days which means that even if buying and processing of RM A starts after 3 days, it will still end the project at the optimal time.

Accordingly it can be concluded that critical activities do not have any float whereas non-critical activities have floats.

Types of CPM

- ✓ Understanding of technical concepts with reference to diagram :

An activity is :

- Ex: In our diagram, 1-2, 3-4, 2-4, 4-5, etc are all activities.

Also, 4-6 is also not an activity.

The beginning and ending point of an activity is called an event. In the network diagram, it is shown as circled numbers.

It is defined as the event which commences an activity.

For activity 1- 2, 1 is the tail event.

For activity 4-5, 4 is a tail event.

It is defined as the event which ends or terminates an activity.

For activity 3-4, 4 is head event.

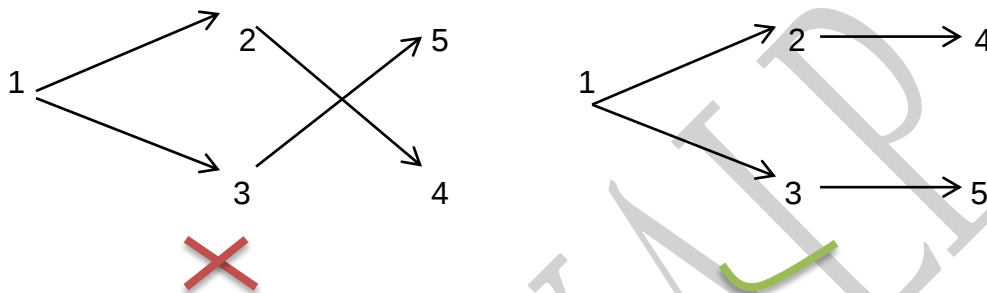
For activity 5-6, 6 is a head event.

Tail Head

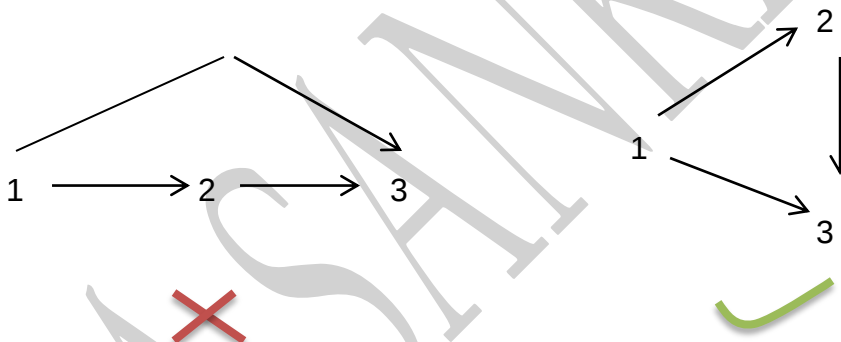
$$1 \longrightarrow 2$$
$$2 \longrightarrow 3$$

✓ General conventions used when making project diagram.

1. Always go from left to right
2. Head events have a higher number than tail events.
3. Avoid criss – crossing or intersection of arrows.



4. Avoid bending of arrows.



5. The arrows lines are not drawn to scale.

Process of critical path network.

The longest path in the network is called as critical path.

ALWAYS REMEMBER TO DRAW DOUBLE LINES TO DENOTE CRITICAL PATH.

TO DENOTE CRITICAL PATH.

1. Each activity is represented by only one arrow in the network.
2. The network diagram is a logical and graphical representative between various activities in a project. Hence length of arrow is immaterial.
3. The flow of a diagram is always from left to right.
4. Arrows should be straight (neither bent nor curved).

5. Arrows should not criss-cross each other until & unless absolutely necessary.
6. Angle between the arrows should be as big as possible.

❖ Slack time:

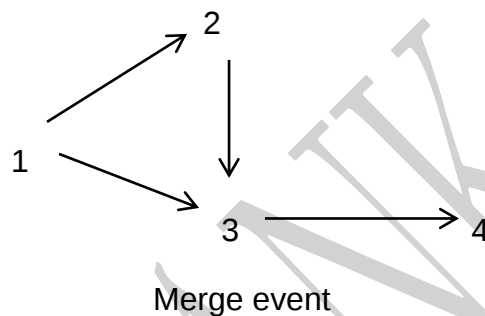
The slack time of an event is the **difference between latest finish time & earlier start time of an event.**

Slack time = difference of E & L

Critical events have zero slack time. Whereas non-critical events have slack time.

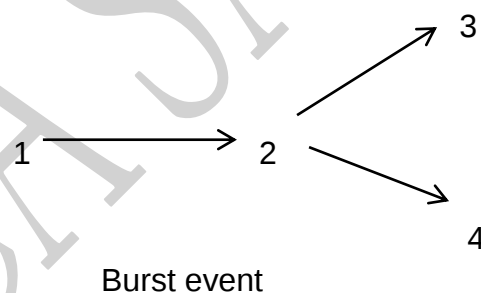
❖ Merge event:

An activity which **represents completions of multiple activities** is called merge events.



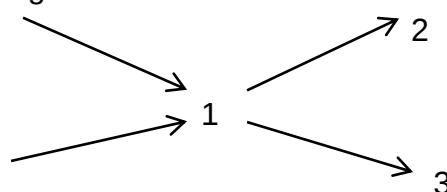
❖ Burst event

An event which **represent start of multiple activities** is called as a burst event.



❖ Merge & burst event:

An event which is merge as well as burst is called merge & burst event.



Meaning of critical activities

Critical activities are the ones which are part of the critical path. Since project completion time is equal to the length of the critical path it is implied that if any critical activity is delayed, project completion will also be delayed i.e. critical activity cannot be delayed.

Non-critical activity

Those activities which are not on critical path non critical activities.

The length of non-critical path is less than that of critical paths.

The difference between length of any non-critical path and critical path represent the margin available for acceptable delay in any of the activities on that non-critical path.

❖ Forward pass

- Forward pass calculation gives the earliest occurrence time or earliest start time of each event in the network.
- This calculation is done from left to right i.e from first event of the project to last event of the project.

❖ Backwards pass.

- Backward pass calculation gives the latest finish time of each event in the network.
- This is done from right to left i.e. from last event to the first event.

❖ Float.

- ✓ Float means the amount of time by which completion of activity can be delayed without delaying the project completion time.
- ✓ Float exist only for non-critical activities whereas there is no float for critical activities.

❖ Dummy activity.

- A dummy activity acts as a connector only for the purpose of completing the network diagram.
- It has no other relevance
- The activity duration of dummy is always 0.

Some of the notation to be used while calculating floats are as follow.

1. ES - earliest occurrence time tail event.
2. LF - latest allowable time of head event
3. EF - ES + duration.
4. LS = LF – duration.

✓ Different types of floats.

1. Total float

It is the amount of time by which completion of an activity can be delayed beyond the earliest possible finish time without affecting the total project completion time.

$$\text{Total float} = \text{LS} - \text{ES} \\ \text{LF} - \text{EF}$$

or

2. Free float

It is the amount of time by which completion of an activity can be delayed beyond the earliest finishing time without affecting the earliest possible start of a subsequent activity.

$$\text{Free float} = \text{Total float} - \text{Head event slack}$$

3. Independent float

It is the amount of time by which start of an activity can be delayed without affecting the earliest start of a subsequent activity, assuming that the preceding activity has finished at its latest finishing time.

$$\text{Independent float} = \text{Free float} - \text{Tail event slack.}$$

4. Interfering float

It is that part of total float which reduces the float of subsequent activities. It is the amount of time by which earliest possible start of subsequent activities will be delayed if activity finishes on latest finishing time.

$$\text{Interfering float} = \text{Total float} - \text{Free float}$$

CRASHING

In CPM analysis, we know that critical path is the longest duration of the project. The total cost of any project is determined by that critical path.

Our role as management accountant is always to ensure that there is continuing benefit to the client. Accordingly if we are able to reduce the total duration of the critical path by reducing the duration of critical activities then ultimately it will result into a lower cost.

However in order to reduce the total time duration we will have to employ some extra resources which will result into an additional cost.

This co-relation of cost benefit analysis is called as “CRASHING”. If the resultant crashing gives a positive cost benefit analysis then we will crash the project otherwise we will not crash the project.

✓ Terminologies associated with crashing.

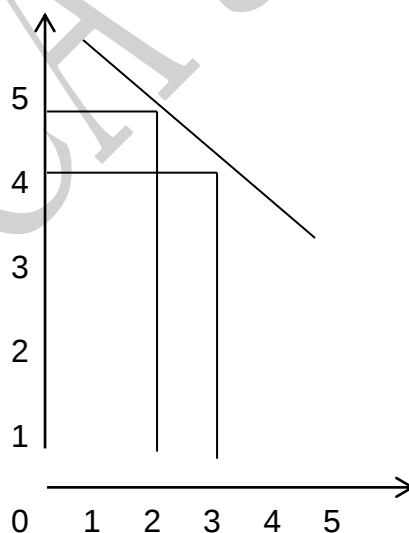
1. Direct cost.

When we estimate the duration of various activities in a project it is assumed that a normal amount of labour or machines will be required to complete these activities.

However, in an intention to accelerate the project additional direct costs like improvement in technology or purchase of machinery is done to ensure that project is completed in minimum possible time.

In short when we want to complete the project in a shorter period than the current critical path, then we will be required to employ more resources & hence the direct cost will increase.

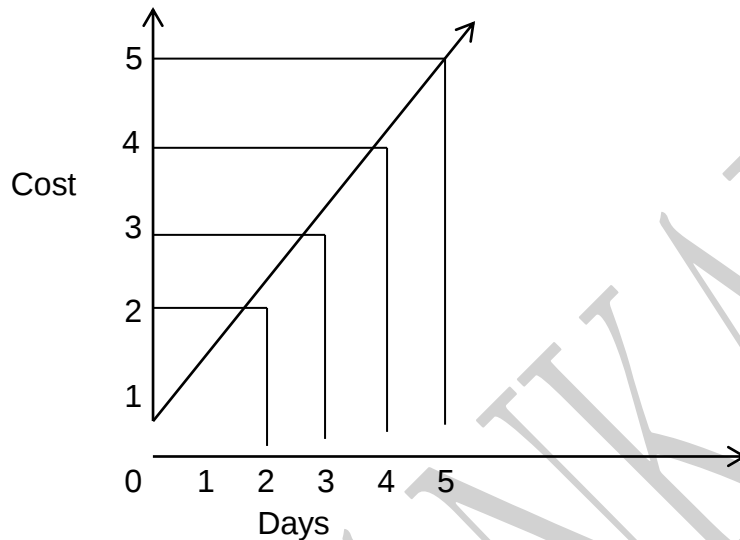
Direct cost line



Ex – If labour is DC, then if no of days is reduced then either more labour will be injected or some technology will be introduced. Hence though no of days will reduce, the cost will increase.

2. Indirect cost.

Indirect cost of an activity refers to the rent & overheads incurred for the project which is dependent on the time duration of the project. Therefore, when we reduce the project completion time the indirect cost on totality basis will also decrease.



Ex - If no of days reduced then indirect cost will also reduce.

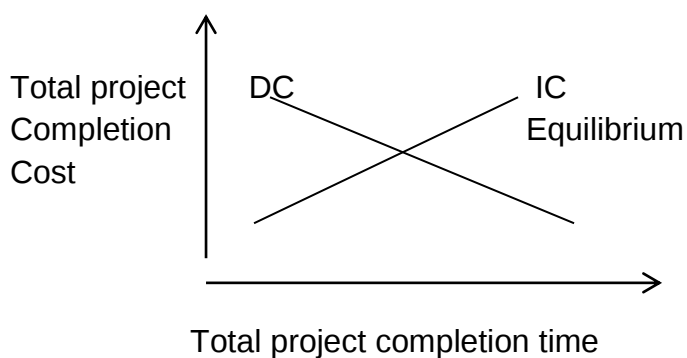
Ex – If portion of class is finished early the cost of rent electricity etc will reduce.

3. Total cost.

The total cost of a project is the summation of direct and indirect costs.

i.e. total cost = Direct cost + Indirect cost

↓ ↓
 Increases Decrease
 If duration decrease



4. Normal time.

The normal time is the activity duration under normal circumstances.

5. Normal cost.

The direct cost associated with normal completion time is called as the normal cost.

6. Crash time.

The minimum time in which an activity can be completed.

7. Crash cost

Cost associated with the crash time of an activity is called as crash cost.

Whenever an activity is crashed the direct cost will increase & indirect cost will decrease.

✓ Rules of crashing

1. Crash activities only on the critical path because crashing the non-critical activities will not result in any cost benefit.
2. If there is more than 1 critical path then both should be crashed together.
3. If all the paths in the project become critical then whole project is to be crashed.
4. Activity on the critical path with least cost slope should be crashed first, keeping the cost benefit analysis in mind.

$$\text{Cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

Cost slope means crashing an activity for a day for a cost i.e. Crash cost per day.

✓ Crashing procedure

1. Draw the network diagram using normal methodology.
2. Find the critical path, project completion time & the various paths associated with the project.
3. Calculate the total normal cost of the project.
Total cost = Direct cost + Indirect cost
4. Prepare a table showing for each activity.
 - a) Cost slope
 - b) Rank in ascending order of cost slope
 - c) Maximum crashable limit.
5. Only critical activity should be crashed & accordingly start crashing the project with the critical activity having lowest rank.
6. Cost computation.

After every crashing, compute the new direct cost, new indirect cost & new total cost. This will help us in finding the total optimal cost.

✓ **Parallel or simultaneous crashing**

1. As we continue to crash the project, the other paths in the project will also be reduced along with the critical path.
2. We may also come across a situation where we may have multiple critical paths. In such a situation, we have to crash all critical paths simultaneously. This can be done in 2 ways.
 - (a) Crash an activity which is common to all critical paths or
 - (b) Crash / activity from each critical path by same duration.

While doing the above procedure.
Always consider the cost benefit analysis
- (c) Continue the crashing procedure till we reach the point of optimal total cost.

PERT.

CPM is a deterministic model which knows the exact activity time based on past duration analysis.

Under **PERT** activity completion time is uncertain & therefore we use a **probabilistic model** to complete the project.

In CPM, we have only 1 time estimate for an activity i.e. we assume that the activity will be definitely complete exactly in that time.

However, in PERT, three possibilities are incorporated for each activity & based on those possibilities the expected duration is worked out.

✓ Three time estimates in PERT are :

1. Optimistic time (To)

- It is the earliest possible completion time of an activity.
- Here, there is an implied assumption that everything happens according to the time. i.e. **No delays, no postponements**.

2. Pessimistic time (Tp)

- It is the maximum possible time taken by an activity for completion.
- An activity will be completed in pessimistic line when there are delays, postponements, lack of communication, lack of coordination, etc.

3. Most likely time (Tm)

- It is the time in which the activity will be completed most often.
- It is neither too optimistic nor too pessimistic.
- Hence most likely time has a value somewhere in between optimistic & pessimistic time.

Based on a statistical probabilistic numerical behavior, a weighted average formula to calculate the expected time is worked out

$$TC = \frac{To + 4Tm + TP}{6}$$

Since the numbers are probabilistic in nature the chance of **variance** & **standard deviation** arises.

The variance between the three time estimates & the expected time is on account of pessimistic and optimistic time estimates.

$$\text{Therefore, Variance} = \left(\frac{Tp - To}{6} \right)^2$$

$$\text{Standard Deviation} = \sqrt{\text{variance}}$$

WHENEVER THERE ARE TWO CRITICAL PATHS, CALCULATE THE STANDARD DEVIATION FOR BOTH AND THEN SELECT HIGHER OF THE TWO.

✓ **Calculation of certain terminologies.**

(a) Variance of an activity = $\left(\frac{T_p - T_o}{6}\right)^2$

(b) Variance of project = variance of critical path
= total variance of critical activities

(c) Standards deviations of project = variance of critical path
= $\sqrt{\text{Total of variance of critical activities}}$

Since this a probabilistic model, it can be used to find out the probability of completing the project in time, completing the project before time or completing the project beyond time.

The formula to calculate that probability is based on a normal distributed table given as below.

$$Z = \frac{\text{Given duration} - \text{expected duration}}{\text{Std deviation of project}}$$